Solutions for the file of Examples 2. pdf

1. The weight of a sophisticated running shoe is normally distributed with a mean of 340g and a variance of 200 g^2.

a) What is the probability that a shoe weighs more than 370g?

$\mu = 340g$
$\sigma^2 = 200\ g^2$

$P(x > 370) = ?$
$P(x > 370) = 1 - \phi(z) = ?$
$$z = \frac{x-\mu}{\sigma} = \frac{370-340}{\sqrt{200}} = 2.12$$
According to the z table the probability by 2.12 is 0.982997.
So:
$\phi(z) = 0.982997$
$P(x > 370) = 1 - 0.982997 = \mathbf{0.0169}$

Thus, 0.0169 is the probability that a shoe weighs more than 370g.

b) What must the standard deviation of weight be in order for the company to state that 99.9% of its shoes are less than 370g?

$\mu = 340g$

$P(x < 370) = 0.999$

$\sigma^2 = ?$

$\phi(z) = 0.999$

According to the z table the z value by 0.999 probability is 3.09.

$$z = \frac{x-\mu}{\sigma} \to 3.09 = \frac{370-340}{\sigma} \to \sigma = 9.71 \to \boldsymbol{\sigma^2 = 94.28\ g^2}$$

c) If the variance remains at 200 g^2, what must the mean weight be in order for the company to state that 99.9% of its shoes are less than 370g?

$\sigma^2 = 200\ g^2$

$\mu = ?$

$P(x < 370) = 0.999$

$\phi(z) = 0.999$

According to the z table the z value by 0.999 probability is 3.09.

$$3.09 = \frac{370-\mu}{\sqrt{200}} \to \boldsymbol{\mu = 326.4\ g}$$

2. The diameter of the dot produced by a printer is normally distributed with a mean diameter of 0.05 mm and a variance of 10^{-4} mm^2.

a) What is the probability that the diameter of a dot exceeds 0.065 mm?

$\mu = 0.05\, mm$
$\sigma^2 = 10^{-4}\, mm^2$

$P(x > 0.065) = ?$
$P(x > 0.065) = 1 - \phi(z) = ?$
$$z = \frac{x - \mu}{\sigma} = \frac{0.065 - 0.05}{\sqrt{10^{-4}}} = 1.50$$
According to the z table the probability by 1.50 is 0.933193.
So:
$\phi(z) = 0.933193$
$P(x > 0.065) = 1 - 0.933193 = \mathbf{0.0668}$

Thus, 0.0668 is the probability that the diameter of a dot exceeds 0.065 mm.

b) What is the probability that a diameter is between 0.04 and 0.065 mm?

$\mu = 0.05\, mm$
$\sigma^2 = 10^{-4}\, mm^2$

$P(0.04 < x \leq 0.065) = P(x \leq 0.065) - P(0.04 < x) = ?$
$P(0.04 < x) = ?$
$P(x \leq 0.065) = ?$
$$z = \frac{x - \mu}{\sigma} \to \frac{0.04 - 0.05}{\sqrt{10^{-4}}} = -1.00$$
According to the z table the probability by -1.00 is 0.137857.
$P(0.04 < x) = 0.137857$
$$z = \frac{x - \mu}{\sigma} \to \frac{0.065 - 0.05}{\sqrt{10^{-4}}} = 1.5$$
According to the z table the probability by 1.5 is 0.933193.
$P(x \leq 0.065) = 0.933193$

$P(x \leq 0.065) - P(0.04 < x) = 0.933193 - 0.137857 = \mathbf{0.795}$

c) In what interval will be the diameter with 99% probability?

$$\mu = 0.05\, mm$$
$$\sigma^2 = 10^{-4}\, mm^2$$

$$P(x_a < x \le x_f) = 0.99$$

$$z = \frac{x - \mu}{\sigma} \rightarrow x_a = \mu - z_{\frac{\alpha}{2}}\sigma \;\; ; \quad x_f = \mu + z_{\frac{\alpha}{2}}\sigma$$

$$P\left(\mu - z_{\frac{\alpha}{2}}\sigma < x \le \mu + z_{\alpha/2}\sigma\right) = 0.99$$

According to the z table the z value (by 0.995 probability) is 2.58.

$$P\left(0.05 - 2.58 \cdot \sqrt{10^{-4}} < x \le 0.05 + 2.58 \cdot \sqrt{10^{-4}}\right) = 0.99$$

$$\boldsymbol{P(0.0242 < x \le 0.0758) = 0.99}$$

3. A manufacturer produces piston rings for an automobile engine. It is known that ring diameter is normally distributed with a standard deviation of 0.001 millimetres. A random sample of 15 rings has a mean diameter of 74.036 millimetres.

a) Construct a 99% two-sided confidence interval on the mean piston ring diameter.

$$\sigma = 0.001\, mm$$

$$n = 15$$

$$\bar{x} = 74.036\, mm$$

$$P(\mu_L < \mu < \mu_U) = 0.99$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \mu_L = \bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \;\; ; \quad \mu_U = \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

$$P\left(\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = 0.99$$

According to the z table the z value by 0.995 probability is 2.58.

$$P\left(74.036 - 2.58 \cdot \frac{0.001}{\sqrt{15}} < \mu < 74.036 + 2.58 \cdot \frac{0.001}{\sqrt{15}}\right) = 0.99$$

$$\boldsymbol{P(74.0353 < \mu < 74.0367) = 0.99}$$

b) Construct a 99% lower-confidence bound on the mean piston ring diameter. Compare the lower bound of this confidence interval with the one in part (a).

$\sigma = 0.001\, mm$

$n = 15$

$\bar{x} = 74.036\, mm$

$P(\mu_a < \mu) = 0.99$

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \mu_a = \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$P\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \ < \mu\right) = 0.99$

According to the z table the z value by 0.99 probability is 2.32.

$P\left(74.036 - 2.32 \frac{0.001}{\sqrt{15}} \ < \mu\right) = 0.99$

$\boldsymbol{P(74.035 \ < \mu) = 0.99}$

4. *The sugar content of the syrup in canned peaches is normally distributed. A random sample of n=10 cans yields a sample standard deviation of s=1.8 milligrams and sample mean 32.4 g.*

a) Calculate a 90% upper confidence limit for expected value of the sugar content.

The unit of the sample mean is incorrect in the text of the example. I will use mg as unit during the solution of the example.

$n = 10 \rightarrow \nu = 10 - 1 = 9$

$s = 1.8\, mg$

$\bar{x} = 32.4\, mg$

$P(\mu \leq \mu_U) = 0.90$

$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \rightarrow \ \mu_U = \bar{x} + t_\alpha \frac{s}{\sqrt{n}}$

$P\left(\mu \leq \bar{x} + t_\alpha \frac{s}{\sqrt{n}}\right) = 0.90$

According to the t table the t value by 0.90 probability and by $\nu = 9$ (one-sided) is 1.383.

$P\left(\mu \leq 32.4 + 1.383 \frac{1.8}{\sqrt{10}}\right) = 0.90$

$\boldsymbol{P(\mu \leq 33.187) = 0.90}$

b) Calculate a 90% upper confidence limit for the variance of the sugar content.

$n = 10$

$s = 1.8\, mg$

$P(\sigma^2 \leq \sigma_U^2) = 0.90$

$$s^2 = \frac{\chi^2 \sigma^2}{\nu} \rightarrow \sigma^2 = \frac{s^2 \nu}{\chi^2} \rightarrow \sigma_U^2 = \frac{s^2 \nu}{\chi_L^2}$$

$$P\left(\sigma^2 \leq \frac{s^2 \nu}{\chi_L^2}\right) = 0.90$$

According to the χ^2 table the χ_L^2 value by 0.90 probability and by $\nu = 9$ is 4.17.

$$P\left(\sigma^2 \leq \frac{1.8^2 \cdot 9}{4.17}\right) = 0.90$$

$$\boldsymbol{P(\sigma^2 \leq 7.0) = 0.90}$$